Solution of smoothing problems with obstacles

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For given sets of indexes $I_0$, $I_1$, $I_0 \cap I_1 = \emptyset$, $I_0 \cup I_1 = I$, obstacles $\varepsilon_i > 0$, $i \in I_1$, pairwise distinct points $X_i \in \mathbb{R}^n$, $i \in I$, and values $z_i \in \mathbb{R}$, $i \in I$, we consider the classical (multivariate) smoothing problem with obstacles

$$\min_{f \in \Omega} \|Tf\|^2,$$

where $\Omega = \{f \in L^2(\mathbb{R}^n) \mid f(X_i) = z_i, i \in I_0, |f(X_i) - z_i| \leq \varepsilon_i, i \in I_1\}$ (see, e.g., [6] or [1]).

The solution of problem (1) is a natural spline. The necessary and sufficient conditions describing the solution are known (see [6]) but finding an algorithm to solve this problem is still an open problem. A quite natural method of adding-removing knots is proposed in [6], but this method can generate a cycle [4].

For any problem with obstacles (1) there exists an equivalent smoothing problem with weights

$$\min_{f \in \Omega_0} \left(\|Tf\|^2 + \sum_{i \in I_1} w_i |f(X_i) - z_i|^2\right),$$

where $\Omega_0 = \{f \in L^2(\mathbb{R}^n) \mid f(X_i) = z_i, i \in I_0\}$, $w_i \geq 0$, $i \in I_1$, see [5]. By the equivalence of problems (1) and (2) we mean that the initial data $X_i$, $z_i$, $I_0$, $I_1$, and the solutions of the problems coincide.

This equivalence allows us to reduce the solution of smoothing problem with obstacles to finding an equivalent smoothing problem with weights. Solving the latter is an easy task: the problem reduces to a linear system of equations.

The equivalence of problems with obstacles and weights in special cases have been studied, e.g., in [2] (univariate case without interpolation knots) and in [1] (multivariate case with positive weights).
We present the equation connecting deviations and weights in the classical case where the weights are non-negative (see [3]). The equation contains an arbitrary symmetric regular matrix as a free parameter. We propose a method for solving this equation and illustrate it by several examples.

This is a joint work with Assoc. Prof. Peeter Oja.

References


