Algebras of analytic functions on infinite dimensional domains
Mini-course October, 2014
University of Tartu

Richard M. Aron Kent State University

General Outline

We plan three lectures on algebras of analytic functions on infinite dimensional domains. The first lecture will be largely review, in which we discuss classical algebras defined on one (or finite) dimensional domains. Initially, we will focus on the two algebras $\mathcal{H}^\infty(D)$ and $\mathcal{A}(D)$ which are, respectively the bounded analytic (= holomorphic) functions from the unit disc $D \to \mathbb{C}$ and the analytic functions $D \to \mathbb{C}$ that are continuous on $\overline{D}$. For either of these algebras, $\mathcal{A}$, we let

$$\mathcal{M}(\mathcal{A}) = \{\varphi : \mathcal{A} \to \mathbb{C} \mid \ \varphi \text{ is a homomorphism}\}.$$

When $\mathcal{A} = \mathcal{A}(D)$, the set of such homomorphisms is not very interesting. However, if $\mathcal{A} = \mathcal{H}^\infty(D)$, then we will see that much more is going on. We will briefly review the 1962 Carleson's *Corona Theorem* as well as the undeservedly lesser known work of I. J. Schark, which preceded Carleson's work by one year. Schark's work gives very useful information about what are called *cluster sets*, which will be defined and discussed. It is interesting to note that Schark's work on cluster sets is definitely weaker than Carleson's when we consider analytic functions on D. However, there are very many situations where the Corona Theorem is either false or unknown, and there is no known situation when Schark's theorem fails.

We will also attempt to answer the natural question: Why should we be interested in $\mathcal{M}(\mathcal{A})$? The point is that when $\mathcal{A}$ is a space of analytic functions on a domain, then $\mathcal{A}$ turns out to be the 'largest' domain to which every such function extends analytically. For this, of course one has to put *analytic structure* on $\mathcal{M}(\mathcal{A})$ (although it isn't likely that we'll reach this).

We then will define the space $\mathcal{H}(U)$ of analytic functions $U \to \mathbb{C}$ where $U \subset X$ is an open subset of a complex Banach space X. We will focus somewhat on the algebra $\mathcal{H}_b(X)$ of entire functions $X \to \mathbb{C}$ that are bounded on each ball in X. It may be somewhat surprising and anti-intuitive that the 'largest' domain to which every $f \in \mathcal{H}_b(X)$ extends analytically includes the bidual X^{**} and sometimes even more! This topic is closely related to what is known as *Arens regularity,* which we will review.

In the final lecture, we hope to discuss the extension of Schark's work to $H^\infty(B_{c_0})$ as well as problems with extending it to $H^\infty(B_X)$ for other Banach spaces X. We hope to convince everyone that (1) the arguments are interesting and are not very hard and (2) there are plenty of reasonable, natural open problems in this area.

Some references

Aron, Richard M.; Carando, Daniel; Lassalle, Silvia; Maestre, Manuel; *Cluster values of holomorphic functions of bounded type*, Trans. A.M.S., to appear.

Aron, Richard M.; Carando, Daniel; Gamelin, T. W.; Lassalle, Silvia; Maestre, Manuel; *Cluster values of analytic functions on a Banach space*, Math. Ann. 353 (2012), no. 2, 293-303.

Aron, R. M.; Cole, B. J.; Gamelin, T. W.; *Weak-star continuous analytic functions*, Canad. J. Math. 47 (1995), no. 4, 67-3683.

Aron, R. M.; Cole, B. J.; Gamelin, T. W.; *Spectra of algebras of analytic functions on a Banach space*, J. Reine Angew. Math. 415 (1991), 51-93.

Aron, R. M.; Galindo, P.; Garca, D.; Maestre, M.; *Regularity and algebras of analytic functions in infinite dimensions*, Trans. Amer. Math. Soc. 348 (1996), no. 2, 543-559.

Carleson, Lennart; *Interpolations by bounded analytic functions and the corona problem*, Ann. of Math. (2) 76 (1962) 547-559.

Schark, I. J.; *Maximal ideals in an algebra of bounded analytic functions*, J. Math. Mech. 10 (1961) 735-746.